

Pulsar Profile construction based on Double-Redundant-Dictionary and Same-Scale L1-Norm Compressed Sensing

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Abstract

Spacecraft navigation in space requires high real-time. Towards this end, when using pulsar navigation, it is necessary to reduce the pulse profile construction time. In this paper, the method of constructing a pulse profile based on compressed sensing (CS) is improved from two aspects: Double Redundant Dictionary (DRD) search method and Same-Scale L1-Norm (SSLN) cost function. The method of calculating the optimal dimension of the DRD is given. A random observation matrix is designed. We provide both analytical and experimental evidence that supports our method. The proposed method is compared with the traditional methods of Amplitude Search (AS) and Hadamard Observation Matrix (HOM) in terms of computational complexity. Simulation using NASA data is consistent with the quantitative analysis. It is illustrated that DRD and SSLN can serve as an effective method with the resulting running time considerably less than that in AS or HOM. The proposed method gets a considerable reduction in pulse profile construction time.

Keywords

Same-Scale L1-Norm (SSLN); Double-Redundant-Dictionary (DRD); X-ray Pulsar; Compressed Sensing (CS); profile construction.

1. Introduction

The use of natural celestial bodies for navigation on the spacecraft has a history of decades [1]. The primary methods are as follows: the first is to determine the attitude and position of the aircraft by detecting celestial bodies in different positions on the celestial sphere [2]; the second is the use of highly stable pulsar radiation signals to estimate the position of the spacecraft [3]. The second method has gained the attention of many space powers as their research focus. For example, Sala et al. analyzed the feasibility of pulsar navigation in a report to the European Space Agency [4]. The United States has implemented a series of plans to explore the method of pulsar navigation and conducted extensive research: One of NASA's tasks from RXTE was to observe X-ray pulsars; Sheikh and Pines proposed a method of correcting the position of a spacecraft and obtained better than 200m accuracy by simulation [1]. The 2016 NASA-funded SEXTANT (The Station Explorer for X-ray Timing and Navigation Technology) program will prove for the first time the feasibility of navigating on pulsar-navigating spacecraft [5].

Pulsar profile construction is the basic work of Pulsar navigation. The traditional method of Pulsar profile construction is the epoch folding: the arrival time of all the photons is converted into the first cycle, and then the first cycle is divided into N bins to calculate the number of photons contained in each bin. This method takes a long time to accumulate and contains noise, which is not conducive to real-time spacecraft navigation. Because the spacecraft is fast, the general speed can be several kilometers per second, so the real-time navigation has a great impact on the accuracy. In order to improve the real-time navigation, some scholars have

introduced compressed sensing (CS) technology that can extract useful data from a large amount of data: Z. Su proposed for the first time the use of CS to construct a pulse profile and verified that this method has a higher signal-to-noise ratio [6]. Then Sheng-liang Li proposes to construct the pulse profile by searching the amplitude [7], which is called Amplitude Search method (AS) for the convenience of narration. Li-rong Shen proposed an observation matrix method based on Hadamard matrix to improve the efficiency of the Compression Sensing method and reduced the calculation time [8], which is called Hadamard Observation Matrix (HOM) method.

Some of the fundamental problems have remained unsolved. One problem is searching for the optimal value requires $N = P/bin$ searches in the above three methods of compressive sensing, where P is the period of the pulsar, bin is the minimum time interval when the epoch is folded. Another problem is L2-norm is used as the cost function. They all take a lot of time to calculate. In order to further reduce computation, we continue to improve the algorithm from two aspects: the establishment of redundant dictionary and recovery algorithm. In this paper, firstly, rough redundant dictionary and exact redundant dictionary are set up to reduce search times and search optimal value more quickly. Then Same-Scale L1-Norm (SSLN) is designed as a cost function to further shorten the calculation time.

The remainder of the article is organized as follows: In the second part, the mathematical model of the photon arrival time of a pulsar is analyzed. In the third part, we optimize the algorithm from three aspects: signal sparsity, random observation and recovery algorithm. The proposed method is compared with AS and HOM.

2. Pulsar Profile Model

The pulsar integral pulse profile can be expressed by Eq. (1):

$$x(t) = b + a \cdot p(t - \Delta t) + \hat{n}(t), n \in \{1, 2, \dots, N\} \quad (1)$$

Where $p(\cdot)$ represents the standard pulse profile, Δt represents the phase of the pulse, that is, the TOA (time of arrival) of the pulse. a is the pulse profile amplitude, $\hat{n}(t)$ represents a random noise, b represents a background noise. The procedure, which recover the pulsar rate function employing the measured photon TOAs, is epoch folding.

The process of epoch folding is as follows: First, assume that the pulsar observation period $T_{obs} \approx N_p \cdot P$, where P is the period of the pulsar. For epoch folding, P is a known constant during observation. Record all photon TOAs (time of arrival), fold them into a period, and divide the period into equal bins which length is T_b . Calculate the photon number of each bin $c(t_i)$, t_i is the intermediate moment of the i -th bin, the empirical probability function can be expressed as:

$$\tilde{\lambda}(t_i) = \frac{1}{N_p T_b} \sum_{j=1}^{N_p} c_j(t_i) \quad (2)$$

3. DRD and SSLN Compressed Sensing

Suppose that an $N \times 1$ compressible discrete signal $\mathbf{X} = \{x_i \in R | i = 1, 2, \dots, N\}$ can be expressed as $\mathbf{X} = \Psi \mathbf{A}$, where Ψ represents a sparse basis or a redundant dictionary in R^N . $\mathbf{A} = \{a_1, a_2, \dots, a_k\}$ is the sparse representation of $\mathbf{X}$ on the Ψ , is the signal representation in the time domain. In sparse domains of Ψ , the signal can be sampled by Eq. (3):

$$\mathbf{y} = \Phi \mathbf{x} + \mathbf{e} = \Phi \Psi \mathbf{A} + \mathbf{e} = \Theta \mathbf{A} + \mathbf{e} \quad (3)$$

Where Φ represents a measurement matrix; Θ represents the sensing matrix; ϵ represents the noise. CS is mainly composed of three parts: the signal sparsity, random observation and recovery algorithm.

3.1. Signal Sparsity Based on DRD

Sparse expression of $\mathbf{X}$ is finding an orthogonal basis Ψ , so that $\mathbf{X}$ can be expressed with a sparse vector $\mathbf{A}$, most elements of which are 0 or very small value, that is to say there are a few non-0 elements. The principle of establishing a redundant dictionary is that the more useful information each element of the dictionary contains, the sparser it can be. It can be seen from Eq. (4) that to establish redundant dictionary of $\mathbf{X}$, we need to analyze $p(\cdot)$. $p(\cdot)$, which is a periodic signal, has a single or multiple peaks, a large part of the zero elements (Photons are not accepted when the pulsar sweeps out of the receiver.) The traditional approach is to create a redundant dictionary as Eq. (4):

$$\Psi = \{\varphi(t) | \varphi(t) = p(t - n\tau)\}, n \in \{1, 2, \dots, N\} \quad (3)$$

Where $p(\cdot)$, which is a standard pulse profile, can be seen as a periodic continuous curve. Since the computer can only process digital signals, suppose the cycle P is evenly divided into N bins using the method of epoch folding, and the length of each bin is τ , The entire standard profile can be traversed through searching N times, so the computational complexity is $O(N)$.

In order to reduce the number of searching and the complexity of the algorithm, this paper designed two redundant dictionaries Ψ_1 and Ψ_2 :

$$\Psi_1 = \{\varphi_1(t) | \varphi_1(t) = p(t - n_1\tau_1)\}, n_1 \in \{1, 2, \dots, N_1\} \quad (4)$$

$$\Psi_2 = \{\varphi_2(t) | \varphi_2(t) = p(t - n_2\tau_2)\}, n_2 \in \{1, 2, \dots, N_2\} \quad (5)$$

Where Ψ_1 is used to search roughly, Ψ_2 is used to search accurately. The sparse expressions of the pulse profiles $\mathbf{X}(n)$ are: $\mathbf{X}(n) = \Psi_1 \cdot \mathbf{A}_1$ and $\mathbf{X}(n) = \Psi_2 \cdot \mathbf{A}_2$ respectively. That is, by setting $\mathbf{A}_1$ and $\mathbf{A}_2$, Ψ_1 and Ψ_2 can be searched for. The complexity of traversing Ψ_1 is $O(N_1)$, and the complexity of traversing Ψ_2 is $O(N_2)$. So the computational complexity of the proposed method is $O(N_1 + N_2)$, the complexity of the traditional method is $O(N)$, $N = N_1 \times N_2$. And, the step length of accurate search $\tau_2 = \tau$, that is to say, is the same as the AS method and the HOM method. The following shows that the proposed algorithm is simpler:

Let

$$\bar{N} = \max(N_1, N_2) \quad (6)$$

$$\underline{N} = \min(N_1, N_2) \quad (7)$$

Then

$$N_1 + N_2 \leq 2\bar{N} \quad (8)$$

And

$$N_1 > 2, N_2 > 2 \quad (9)$$

Then

$$2\bar{N} < \bar{N} \cdot \underline{N} = N_1 \cdot N_2 \quad (10)$$

So

$$O(N_1 + N_2) < O(N) \quad (11)$$

In this paper, the optimal N_1 and N_2 values are calculated by the following method:

Because

$$N_1 \cdot N_2 = N (N \text{ is a constant, } N_1, N_2, N \in (0, +\infty)) \quad (12)$$

So

$$N_1 + N_2 \geq 2\sqrt{N_1 \cdot N_2} = 2\sqrt{N} \quad (13)$$

So

$$\min(N_1 + N_2) = 2\sqrt{N} \quad (14)$$

This paper assumes that $bin = 10\mu s$, the period of pulse data $P = 33663.8\mu s$, $N = 3367$, so $N_1 = N_2 = 59$. As can be seen from Figure 1, the profile of the rough search redundant dictionary can generally reflect the characteristics of the pulsar profile: because it contains less noise, it can achieve the desired effect with only a simple filtering and can be used as a rough search redundant dictionary. Figure 2 shows that the redundant dictionary profile for accurate search contains more noise and can be smoothed after multiple filtering. It can also serve as a redundant dictionary.

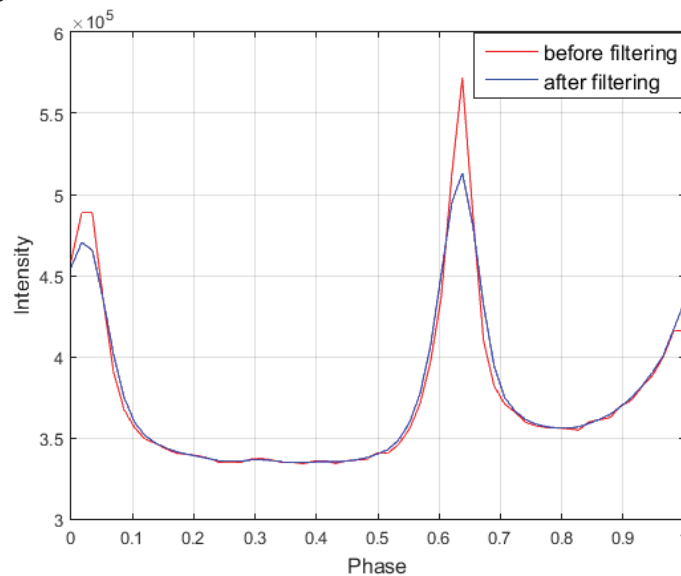


Figure 1. Profile of rough search

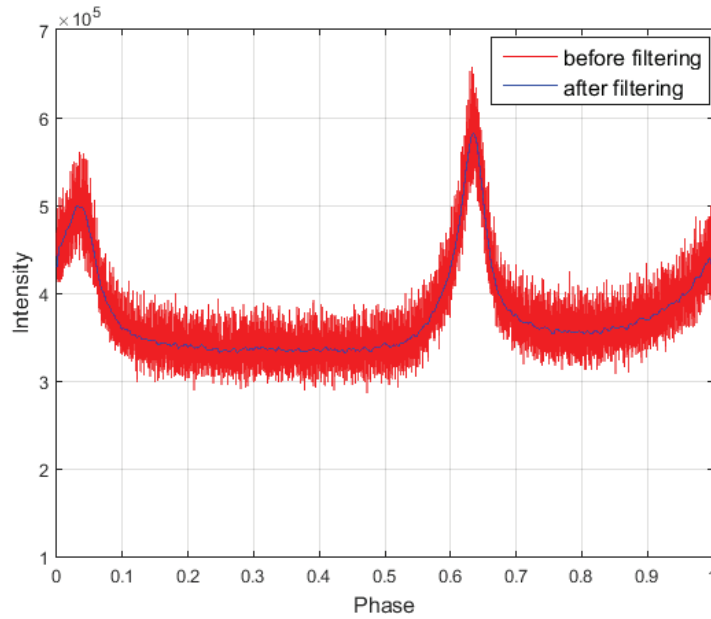


Figure 2. Profile of accurate search

3.2. Random Observation

The random observation of the $\mathbf{x}$ signal is how to design the redundant dictionary Ψ which is not related to the observation matrix Φ , so that the $\mathbf{x}$ information will not be further destroyed. In fact, we must consider the matrix is easy to achieve.

The function of the observational matrix of AS method is to select the columns of redundant dictionary, and then search the redundant dictionary by changing the observation matrix. The computational complexity of this method is $O(N)$.

According to Eq. (3), in principle, when constructing the observation matrix Φ , it is necessary to reduce the number of samples and ensure the signal recovery. Scholars have done some research in this area. The observation method of AS actually sparsely represents the curve. The HOM method designs an observation matrix optimized based on Hadamard matrix and analyzes the effect of sampling rate on the contour construction.

In this paper, a more concise observation matrix is designed with less computation. Assume that the sampling rate is $R_s = M/N$, the sample size is M , and the total number of signals is N . First, we select M random integers with different numbers between 1 and N , and arrange them from small to large to obtain the M -dimensional vector $\mathbf{E}$. The $(i, \mathbf{E}_i)$ element of the $N \times M$ observation matrix Φ is 1, and the remaining elements are all 0, where e_i represents the i th element of $\mathbf{E}$. That is, each row of Φ has only one element, and each column has at most one element. Any two rows of Φ are orthogonal. This will ensure that the observed value $\mathbf{y}$ will not be further lost to the sampling information of $\mathbf{x}$, and the number of rows of Φ is determined according to the number of samples. It is equal to the rows of $\mathbf{y}$. Pulsar detection can be viewed as random sampling from the discrete integral profile, and the x-ray pulsar profile observation is $y(m) = \Phi \cdot x(n)$. Compared with the traditional method, the observation vector is reduced from N dimension to M dimension, that is, the observation matrix reduces the influence of search computational complexity from $O(N)$ to $O(M)$. For comparison with the HOM method, the sampling rate is assumed to be 0.6.

3.3. Recovery Algorithm based on SSLN

The signal recovery algorithm is how to quickly and accurately recover the signal. Although the amount of computation of the L1-norm as a cost function is less than the L2-norm cost function of the AS and HOM methods, solving the L1-norm between the standard pulse profile and the

integral pulse profile may encounter unsolvable conditions. As shown in Figure 3, when the lowest point of the blue standard curve is higher than the highest point of the cumulative curve, L1-norm is a constant value according to (18). Where $\tilde{\mathbf{x}}$ represents the standard profile, and $\mathbf{x}$ represents the integral profile. So using L1-norm method can't calculate the optimal solution.

$$\|\tilde{\mathbf{x}} - \mathbf{x}\|_{l1} = \sum_{i=1}^N |\tilde{x}_i - x_i| = \sum_{i=1}^N \tilde{x}_i - \sum_{i=1}^N x_i, \tilde{x}_i > x_j, i, j \in [1, N] \quad (15)$$

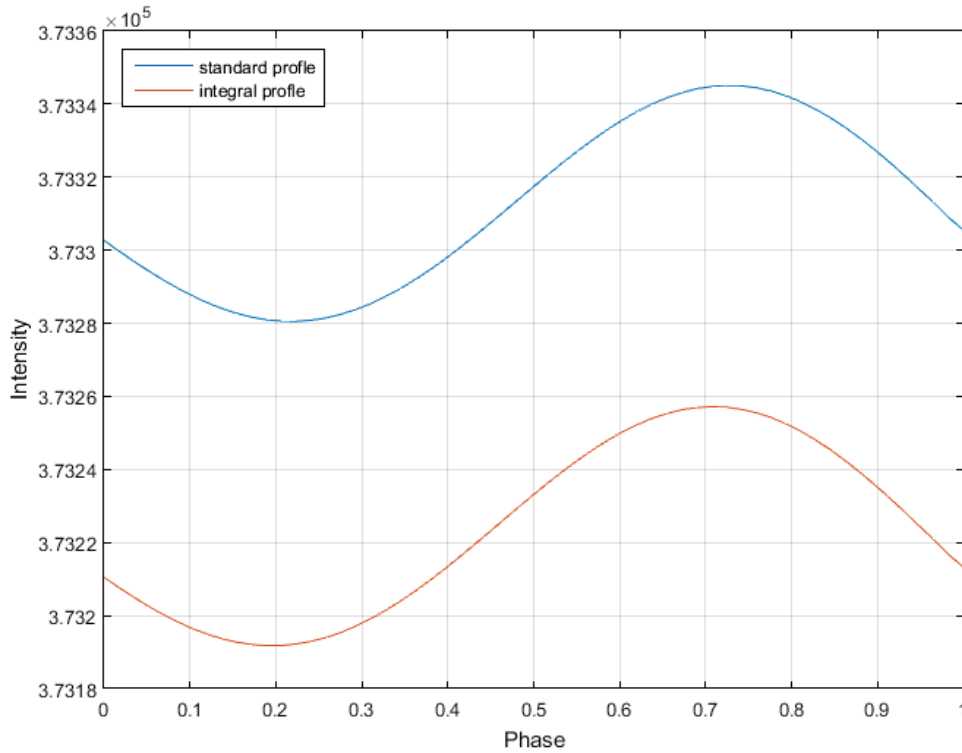


Figure 3. Schematic diagram when the standard and integral curves can't be solved using L1-norm method

In order to avoid this situation, we set a parameter $\eta = \text{mean}(\tilde{\mathbf{x}}) / \text{mean}(\mathbf{x})$ on the basis of 1-norm solution to convert $\tilde{\mathbf{x}}$ and $\eta\mathbf{x}$ to the same scale, and then solve for $\arg \min_{l1} (\|\tilde{\mathbf{x}} - \eta\mathbf{x}\|_{l1})$ to get the current phase value.

Figure 4 shows the correlation between the phase difference and the deviation obtained using SSLN as the cost function. When the phase difference is 0 at the beginning, the deviation is almost 0. As the phase difference increases, it will gradually increase, and reach the maximum when the phase difference is half a cycle, and then continue to decrease. Finally, when the phase difference reaches nearly one cycle, it is gradually approaching 0.

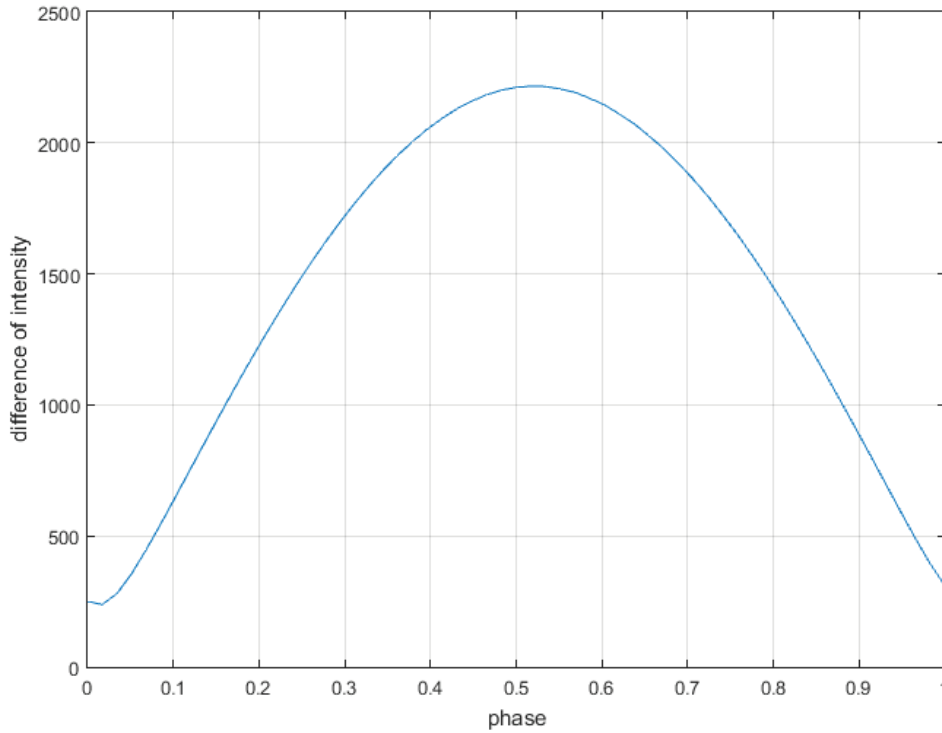


Figure 4. Deviations and phase difference of integral profile and standard profile

Profile recovery algorithm is how to quickly and accurately recover the signal. The AS method uses a combination of search amplitude and phase to determine the amplitude and then search the phase value of the integral profile concluded by the L2-norm cost function. The phase of the pulse is discretely expressed according to Eq. (1), and the relationship between the standard pulse profile and the integral pulse profile can be expressed as:

$$x(t) = b + a \cdot p(t - n\tau) + n(t) \quad (17)$$

Where τ represents the time delay, which is generally equal to the length of the bin; $n\tau$ represents the arrival time (TOA) of the pulse; a represents the amplitude factor; $n(t)$ represents the random noise; b represents the background noise, which can be considered as a constant. As can be seen from Eq. (14), the integral pulse profile can be considered as the standard pulse profile with periodic and amplitude conversion.

The AS method first selects a column of the redundant dictionary and then finds the minimum value by the L2-norm:

$$r_{i,j} = \|\mathbf{y} - \boldsymbol{\theta}_{m \times n} \mathbf{a}_n\| \quad (18)$$

where $i \in [0, N_a]$ represents the number of steps of the search amplitude and $j \in [1, N]$ represents the current column of the redundant dictionary. Assuming that the minimum value of a_n is $a_{\min}$, the maximum value is $a_{\max}$, and the search step size is Δa , the computational complexity is $O(N_a) = O((a_{\max} - a_{\min}) / \Delta a)$. Then select the redundant dictionary of the next column, so traversing redundant dictionary of the computational complexity is $O(N)$. Using the L2-norm as a cost

function, the computational complexity is $O(2N-1)$. The total computational complexity of the AS method is $O(N_a \cdot N \cdot (2N-1))$.

Compared with the AS method, the HOM method omits the amplitude search part. The role of the sparse matrix $\mathbf{A}_{N \times 1}$ is to select the redundant dictionary $\mathbf{\Psi}_{N \times N}$ column, the computational complexity of traversing matrix $\mathbf{\Psi}_{N \times N}$ is $O(N)$. The function of the random observation matrix $\mathbf{\Phi}_{M \times N}$, ($M < N$) is to further select the data and reduce the $\mathbf{y}$ dimension from N to M . The computational complexity of using the L2-norm as a cost function is $O(2M-1)$. The total computational complexity is $O(N \cdot (2M-1))$.

The method proposed in this paper further reduces the computational complexity when traversing redundant dictionaries. According to Eq. (15), the computational complexity is $O(2\sqrt{N})$. Similar to the HOM method, random observations are used to change the dimension from N to M , and the computational complexity of SSLN as the cost function is $O(2M-1)$. The total computational complexity is $O(2\sqrt{N} \cdot (2M-1))$. Although the computational complexity of the Signal sparsity and random observation proposed in this paper is the same as that of the HOM method, a computationally intensive square root operation is calculated for the L2-norm and SSLN is not calculated.

In summary, the complexity of the above three compression sensing methods can be compared in this paper, as shown in Table 1.

Table 1. Comparison of various CS computational complexity.

Methods	CS			Overall complexity
	Signal sparsity	Random sampling	Recovery algorithm	
AS	$O(N_a)$	$O(N)$	$O(2N-1)$	$O(N_a \cdot N \cdot (2N-1))$
HOM	1	$O(2M-1)$	$O(N)$	$O((2M-1) \cdot N)$
DRD&SSLN	1	$O(2M-1)$	$O(2\sqrt{N})$	$O((2M-1) \cdot 2\sqrt{N})$

4. Simulation

In order to verify that the DRD method and the SSLN method are effective and their performance is analyzed, three compression sensing methods (AS, HOM and DRD) and three cost functions (L2-Norm and SSLN) are respectively combined to simulate. After simulation, we got Table 2. The number of times the AS method needs to search for the amplitude varies depending on the data. In order to compare the run time of various methods, it is assumed that the AS method only needs to search for amplitude once. It can be seen from Table 2 that search time is 66%, 55% and 54% higher than the HOM method, respectively, because there is no further sparse sampling. So we can see that the compression sampling has a great effect on the search time. However, the run time of the DRD method proposed in this paper is only 5.3%, 4.1% and 5.9% respectively compared with the AS method, and it is 9.0%, 6.3% and 9.1% respectively compared with the HOM method. It can be seen that the proposed method greatly improves the search efficiency.

In addition, by comparing the different cost functions of the same search method, it can be seen that the running time of SSLN method in the AS method is 38.75% of the L2-norm method. In the HOM method, the running time of the SSLN method is 41.67% of the L2-norm method. In DRD method, the running time of the SSLN method with the same scale is 29.53% of the L2-norm method. In the same way, the L2-norm saves time compared with the standard L1-norm as a cost function. The DRD method is the most time-saving when using the same cost function. Overall, DRD with the same scale L1-norm real-time is the best.

Table 2. Comparison of three CS methods with three cost functions in time consumption

CS methods + Cost function	Traversing steps	Sampling Rate (%)	Pulse Profile Construction Time (S)
AS + L2-norm	3367	100	0.80
AS + SSLN	3367	100	0.31
HOM + L2-norm	3367	60	0.48
HOM + SSLN	3367	60	0.20
DRD + L2-norm	118	60	0.043
DRD + SSLN	118	60	0.0127

5. Conclusion

In this paper, the method of CS to construct the pulsar profile is studied: a search method of the DRD and the cost function of SSLN is proposed, and the computational complexity of various algorithms is analyzed. In this process, we see that when the sampling rate is 0.6, the computational complexity of the DRD method is 2.1% of the AS method and 3.5% of the HOM method. The simulation results using NASA data show that the run time using the DRD method is only 5.4% of the AS method and 9.0% of the HOM method, and the DRD method with the SSLN cost function runs about 1.59% of the AS method, 2.65% of the HOM method. The result of simulating RXTE data by MATLAB shows that the running time of the DRD method with a SSLN cost function is 1.59% of the Amplitude Search method and 2.65% of the HOM method. Comparing the consumption time of various methods, we prove that the DRD method and SSLN cost function has better real-time performance than AS method and HOM method.

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