

Structural Reliability Analysis Based on NATAF Transform and Particle Swarm Optimization

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Abstract

In this paper, a reliability analysis method based on NATAF transform and particle swarm optimization (PSO) is proposed to solve the problem of reliability analysis in which variables are not normally distributed and have correlation. Firstly, the mathematical model of the reliability index is established. Then the Nataf transformation principle is introduced to deal with the correlation between the structural parameters, and finally, the particle swarm optimization algorithm is used for the optimization calculation. The validity and feasibility of the proposed method are verified by the reliability analysis of numerical examples and engineering examples of anti-roll torsion bars.

Keywords

Correlation; NATAF transformation; PSO; Reliability analysis; Nataf transformation.

1. Introduction

In the field of structural reliability analysis, probabilistic models are usually used to describe the uncertainties of load, material properties and installation errors. Based on probability model, a series of effective reliability analysis methods have been developed. Such as first order reliability method (FORM), second order second reliability method (SORM), Monte Carlo method (MCS) and so on. In recent years, in the field of structural reliability analysis, in order to solve practical engineering problems, many scholars put forward many new calculation methods. Meng et al. [1] proposed a new modified control theory algorithm to solve the problem of oscillation in the iterative process, aiming at the phenomenon of chaos, oscillation and periodic solution in the case of a high degree of nonlinearity of the functional function. This method has high efficiency and is relatively stable. Qiu et al. [2] proposed a two-parameter optimization design point reliability analysis method to solve the reliability analysis problem of interval variables in functional functions. Meng et al. [3] proposed a hybrid reliability analysis method based on dimensional reduction algorithm, which was used to solve the mixed uncertainty problem of structures with both fuzzy variables and random variables in engineering practice. Chen et al. [4] improved the Monte Carlo method on the basis of neural network, and reduced The Times of analysis on the premise of ensuring accuracy. Zhao et al. [5] proposed an effective proxy model method for the complex implicit multi-probability constraint conditions in structural reliability design. Xiao et al. [6] proposed a structural reliability analysis method based on sequential iteration model, which has good accuracy and robustness, in view of the fact that the performance function of components or systems in engineering is usually in the form of implicit function. However, today's reliability calculation methods are mostly based on the assumption that the random variables influencing the

structure are independent of each other, and the influence of the correlation of random variables on the reliability index is rarely considered. If the parameter uncertainty and the correlation between variables are ignored, the calculation of the reliability index will bring errors [7]. At present, the reliability analysis methods dealing with correlation mainly include Rosenblatt transform, orthogonal transform and Nataf transform. The Rosenblatt transformation method needs to know the joint distribution function of each random variable. In engineering practice, it is difficult to obtain the joint distribution function, which limits the application of this method. In addition, the orthogonal transformation method proposed by Li has been proved to have some limitations in later studies. For some special distribution types, the correlation coefficient of variables will change after conversion [8].

Based on this, this paper proposes a reliability analysis method based on NATAF transform and BAT algorithm to solve the reliability analysis problem with correlation between variables. This method takes into account the influence of the change of correlation coefficient after the conversion of non-normal variable to normal variable on the reliable index, and particle swarm optimization algorithm has a better optimization effect for the structural functional function with strong nonlinear, and does not need to solve the gradient of the functional function, which improves the calculation efficiency.

2. The Establishment of Reliability Index

Let the functional function of the structure function be $Z = g(X_1, X_2, \dots, X_n)$, where $X_1, X_2, \dots, X_n$ is the basic random variable affecting the structure function.

Let a variable in X , such as X_i be a non-normal distribution variable, with a mean of X , a standard deviation of σ_{X_i} , a probability density function of $f_{X_i}(x_i)$, and a cumulative distribution function of $F_{X_i}(x_i)$. The non-normal variables were normalized by Rackwitz-Fiessler method, and the mean value μ'_{xi} , standard deviation σ'_{xi} and reliable index β of the normal distribution were obtained.

$$\mu'_{xi} = X_i^* - \Phi^{-1}[F_{X_i}(x_i^*)]\sigma'_{xi} \quad (1)$$

$$\sigma'_{xi} = \frac{\varphi\{\Phi^{-1}[F_{X_i}(x_i^*)]\}}{f_{X_i}(X_i^*)} \quad (2)$$

$$\beta = \left\{ \sum_{i=1}^n [(X_i^* - \mu'_{xi}) / \sigma'_{xi}]^2 \right\}^{1/2} \quad (3)$$

β is regarded as the function of the limit state surface point $p(X_1, X_2, \dots, X_n)$, and the minimum value of β is found by solving, namely, the reliable index β and the checking point $p^*(X_1^*, X_2^*, \dots, X_n^*)$ are obtained. From the above, the following optimization mathematical model can be established.

$$\min \quad \beta^2 = \sum_{i=1}^n [(X_i^* - \mu'_{xi}) / \sigma'_{xi}]^2 \quad (4)$$

$$s.t \quad Z = g(X_1^*, X_2^*, \dots, X_n^*) = 0 \quad (5)$$

In order to transform the above constrained mathematical model into an unconstrained mathematical model, the augmented multiplier method is used.

$$\min M_{\lambda}(X^*, r, \lambda) = M(X^*) + \frac{r}{2}[g(X^*)]^2 + \lambda g(X^*) \quad (6)$$

In the formula, $M(X^*)$ is β^2 ; The penalty item is the second item at the right end, and the penalty factor is r . The multiplier term is the third term on the right, and r is the Lagrange multiplier.

3. NATAF transformation

Let's say we have n-dimensional standard normal random variable $Y = (Y_1, Y_2, \dots, Y_n)^T$. The correlation coefficient matrix of Y is $\rho_0 = (\rho_{0ij})_{n \times n}$, and the corresponding joint probability density function is given as follows.

$$\varphi_n(y, \rho_0) = \frac{1}{\sqrt{(2\pi)^n \det(\rho_0)}} \exp\left[-\frac{1}{2} y^T \rho_0^{-1} y\right] \quad (7)$$

$\det(\cdot)$ represents the determinant of the matrix, and $(\cdot)^{-1}$ represents the inverse of the matrix.

N-dimensional random variable $X = (X_1, X_2, \dots, X_n)^T$ is defined, and its correlation coefficient matrix is $\rho = (\rho_{ij})_{n \times n}$. According to the principle of equal probability transformation, the relationship between variable X and variable Y can be obtained as follows.

$$x_i = F_{X_i}^{-1}[\Phi(y_i)], \quad i = 1, 2, \dots, n \quad (8)$$

$F_{X_i}(\cdot)$ is the edge cumulative distribution function of variable X_i . $F_{X_i}^{-1}(\cdot)$ is the inverse of $F_{X_i}(\cdot)$. According to Nataf distribution theory, the joint probability density function of variable X can be deduced by using the derivative rule of implicit function.

$$f_X(x) = \frac{f_{X_1}(x_1)f_{X_2}(x_2)\dots f_{X_n}(x_n)}{\phi(y_1)\phi(y_2)\dots\phi(y_n)} \phi_n(y, \rho_0) \quad (9)$$

$f_{X_i}(\cdot)$ is the edge probability density function of variable X_i , and the probability distribution model of variable X is generally called Nataf distribution.

The correlation coefficient ρ_{ij} of variable X has the following relationship with the correlation coefficient ρ_{0ij} of the equivalent standard normal variable Y

$$\rho_{ij} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \frac{x_j - \mu_{x_j}}{\sigma_{x_j}} \phi_2(y_i, y_j; \rho_{0ij}) dy_i dy_j \quad (10)$$

4. Particle Swarm Optimization

PSO algorithm started from the study of bird feeding state, so that each particle in the n-dimensional search area has two vectors, x_i representing position and v_i representing speed. In each iteration, the particle will learn its own historical best position and exchange it with the population, so as to search and update information [9]. Let $V_i(V_i^1, V_i^2, \dots, V_i^N)$ be the velocity vector of the i th particle and $X_i(X_i^1, X_i^2, \dots, X_i^N)$ be the position vector of the i th particle. The two vectors of the particle can be updated by the following formula.

Mathematical model of velocity vector is updated by

$$v_i(k+1) = v_i(k) + c_1 r_1 (pBest_i(k) - x_i(k)) + c_2 r_2 (gBest(k) - x_i(k)) \quad (11)$$

Mathematical model of position vector is updated by

$$x_i(k+1) = x_i(k) + v_i(k+1) \quad (12)$$

$pBest_i$ is the best location in the history of particle i , $gBest$ is the best position in the history of the group. k is the best position in the history of the group, c_1 and c_2 are learning factor, r_1 and r_2 are random numbers from 0 to 1.

5. Example

5.1. Numerical Example

The limiting state equation is given as follows

$$g = x_2^2 - 2x_1 \quad (13)$$

There is a positive correlation between the random variables x_1 and x_2 , And the correlation coefficient $\rho_{x_1 x_2} = 0.5$, x_1 obey the extremum I type distribution, Its mean and standard deviation are $\mu_{x_1} = 20$ and $\sigma_{x_1} = 5$. x_2 follows a lognormal distribution. Its mean and standard deviation are respectively $\mu_{x_2} = 10$ and $\sigma_{x_2} = 2$. Particle swarm method, FOSM and MCS are used to calculate the reliability indexes. The reliable index results calculated by the three methods are shown in Table 1. The iteration diagram of reliability index and failure probability of NATAF + PSO is shown in Figure 1. The iteration diagram of reliability index and failure probability of NATAF + FOSM is shown in Figure 2.

Table 1. Reliability index calculation results of the three methods.

methods	reliabilityindex	failureprobability	iterative times
MCS	2.6983	0.0034	107
NATAF+FOSM	2.6859	0.0036	6
NATAF+PSO	2.6943	0.0035	4

As can be seen from Table 1, the reliability index calculated by the algorithm in this paper is 2.6943. The reliable index calculated by NATAF+FOSM method was 2.6859. The reliability index calculated by MCS method is 2.6983, and the reliability index calculated by the three

algorithms is roughly the same, which verifies the feasibility of the proposed algorithm. At the same time, the proposed algorithm only needs to iterate 4 times to obtain the reliability index, while the FOSM method needs to iterate 6 times, so the proposed algorithm is better.

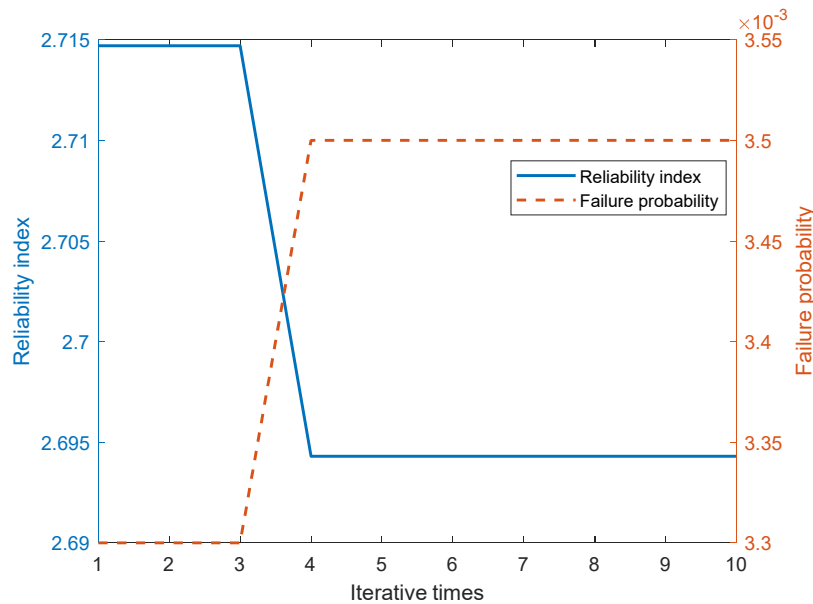


Figure 1. Iteration diagram of reliability index and failure probability

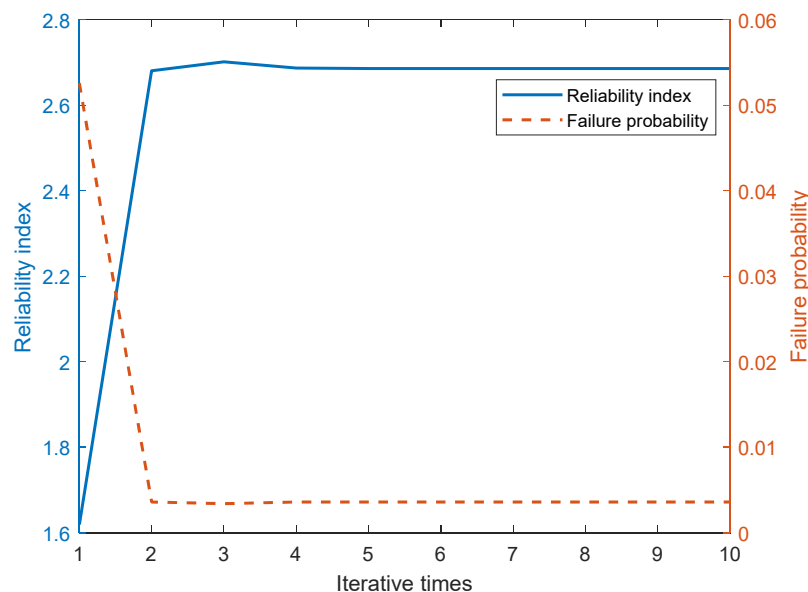


Figure 2. Iteration diagram of reliability index and failure probability

5.2. Engineering Example

Due to lateral rolling torsion bar under high torque for a long time, the structure is safe for driving safety. In order to further validate the effectiveness of the proposed method, taking a bullet in the anti roll torsion bar as the research object, on the reliability analysis, anti roll torsion bar installed on the bogie frame, composed of torsion bar shaft, reverse arm and connecting rod, anti roll torsion bar of the geometric model is shown in Figure 3. The car body transfers the force from the connecting rod through the torsion arm to the torsion bar shaft through the rubber joints on the connecting rod, and then generates torque on the torsion bar shaft. The force diagram of anti-roll torsion bar is shown in Figure 4 [10].

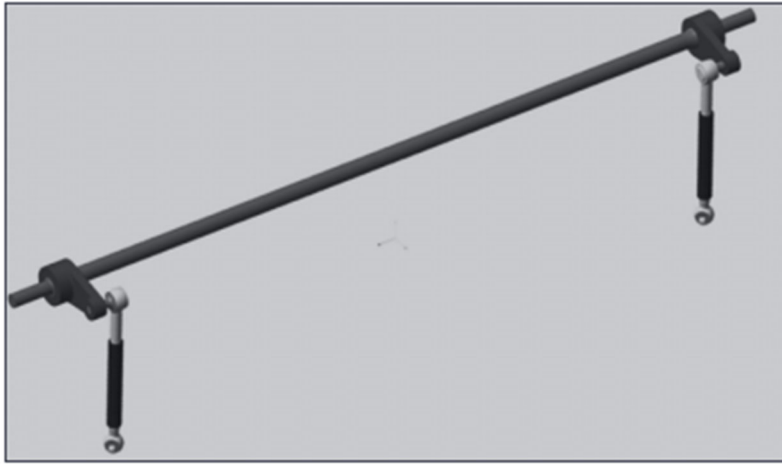


Figure 3. Geometric model of anti-roll torsion bar

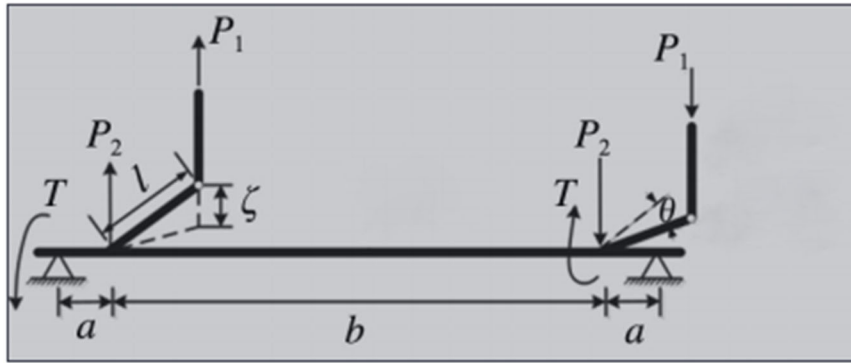


Figure 4. Schematic diagram of force of anti-roll torsion bar

The possible failure modes of anti-roll torsion bar include bending strength failure, torsional strength failure and combined bending and torsion failure, which together affect the reliability of anti-roll torsion bar structural system. The allowable shear stress of torsion bar and shaft materials is $[\tau] = 700 \text{ MPa}$, the tensile limit $[\sigma_b]$ is 1450MPa, and the yield limit $[\sigma_s]$ is 1300MPa. The load P follows the lognormal distribution, with the mean of $\mu_p = 34800$ and the standard deviation of $\sigma_p = 18$. The diameter of torsion bar $d(\text{mm})$ follows the normal distribution $N(35, 3)$, and P is positively correlated with d , and the correlation coefficient is 0.2.

Reliability analysis when considering torsional failure

When the structure has torsional strength failure, its maximum torque is $T_{\max}$. When the structure has torsional strength failure, its torsional section coefficient is W .

$$T_{\max} = pl \quad (14)$$

$$W_p = \pi d^3 / 16 \quad (15)$$

Where l is the length of the torsion arm, and its value is 240mm.

When torsional strength failure occurs to the structure, its functional function is expressed as follows.

$$g_1 = [\tau] - \frac{T_{\max}}{W_p} = 700 - \frac{240 \times x_1}{\pi x_2^3 / 5} \quad (16)$$

By applying the method in this paper, the reliability index can be obtained through calculation. The detailed calculation results are shown in Table 2. The reliability index iteration process is shown in Figure 5.

Table 2. Reliability index calculation results

Reliabilityindex	Failureprobability	Functional value
4.3643	6.3772e-6	0.0373

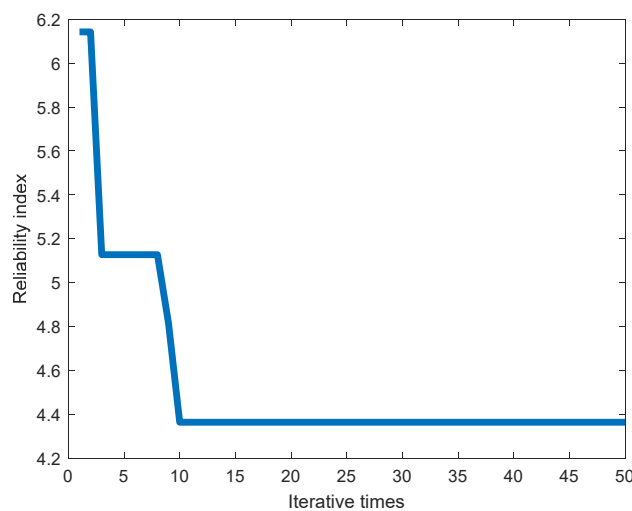


Figure 5. Iteration diagram of reliability index in torsional failure

It can be seen from Table 2 that the failure probability is close to 0, indicating that this calculation example has high reliability under the conditions in this paper. At the same time, it can be seen from Figure 5. that the algorithm in this paper has a fast convergence rate and can quickly obtain a feasible solution.

(2) Reliability analysis when bending strength failure is considered

When the bending strength failure occurs, the maximum bending moment is M and the torsional section coefficient is W_z . The type of $a = 110$ mm.

$$M_{\max} = pa \quad (17)$$

$$W_z = \pi d^3 / 32 \quad (18)$$

When the bending strength failure occurs to the structure, its functional function is expressed as follows.

$$g_2 = [\sigma_b] - \frac{M_{\max}}{W_z} = 1300 - \frac{110 \times x_1}{\pi x_2^3 / 32} \quad (19)$$

By applying the method in this paper, the reliability index can be obtained through calculation. The detailed calculation results are shown in Table 3. The reliability index iteration process is shown in Figure 6.

Table 3. Reliability index calculation results

Reliabilityindex	Failureprobability	Functional value
1.9410	0.0261	0.0975

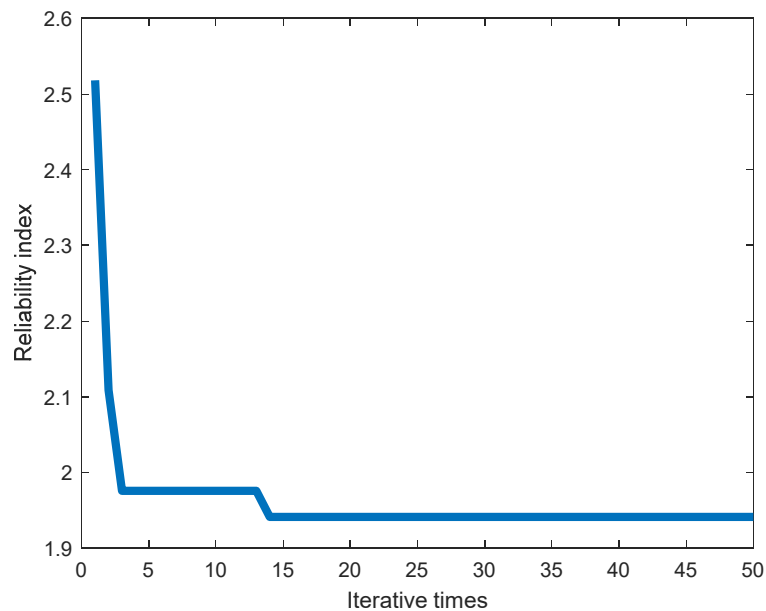


Figure 6. Iteration diagram of reliability index in failure of bending strength

It can be seen from Table 3 that the failure probability is close to 0, indicating that this calculation example has high reliability under the conditions in this paper. At the same time, it can be seen from Figure 6 that the present algorithm in this paper has a fast convergence rate and can quickly obtain a feasible solution.

6. Conclusion

In this paper, a reliability analysis method based on NATAF transform and particle swarm optimization (PSO) is proposed to solve the reliability analysis problem in which the variables in the functional function and there is correlation between the variables. A numerical example is given to verify the feasibility of the proposed method. The results show that the proposed method has fewer iterations compared with the traditional algorithm and improves the computational efficiency. Two failure modes of the anti-roll torsion bar are considered in the engineering example, and the reliability indices are obtained by using the proposed algorithm. The failure probabilities are close to 0, which indicates that the anti-roll torsion bar model structure is reliable.

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